Sborník vědeckých prací Vysoké školy báňské - Technické univerzity Ostrava
řada stavební, ročník 15, číslo 2, rok 2015
článek č. 9

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RELIABILITY ANALYSIS OF THIN-WALLED CYLINDRICAL SHELLS

Abstract

The subject of the article is the verification of the reliability of thin-walled rotationally symmetric cylindrical shells, using probabilistic approaches. Internal forces and stress of the shell are analysed assuming a membrane action. Material and geometric characteristics of the steel shell are considered as random variables. The reliability index is evaluated using the Latin Hypercube Sampling method. The results of the reliability analysis are derived in a general form, so that they may be useful for assessing the reliability of tanks and pipes.

Keywords

Reliability, safety, probability, steel, tank, course, shell, thickness.

1 INTRODUCTION

Apparently, rotationally symmetric shells represent the most frequently encountered type in engineering practice. If internal moments are absent and only inner forces are present in the shell, then the shell is said to be in the so-called membrane state. This state is advantageous and desirable in terms of material strain, and it is thus important in design to ensure that this state is compromised minimally by unsuitable design [2]. Vertical cylindrical steel storage tanks represent a typical example of rotationally symmetric shells. A rotationally symmetric tank is loaded by hydrostatic pressure constantly along parallel circles. The solution of this type of shell is relatively simple, and in many cases, it may be applied for the reliability analysis of pressure pipes of power plants.

2 ROTATIONALLY SYMMETRIC SHELLS IN THE MEMBRANE STATE

The evaluation of internal forces and stresses in rotationally symmetric tank shapes can be simplified and calculated as rotationally symmetric thin shells, see Fig. 1. The derivation of the computational formulas for determining the internal forces of the shell proceeded particularly from [4].

The rotationally symmetric thin shell depicted in Fig. 1. is characterized by the prescription of its middle surface and thickness t. If we project a perpendicular (normal) n to the mid-surface from point M, we can use this normal to define two mutually perpendicular planes which intersect the mid-surface at the points of maximum and minimum curvatures. These planes are called the principal normal planes.

The first principal plane is determined by the normal n and axis of rotation o. The normal n is at an angle α with the axis of rotation. The intersection of the first principal plane with the shell mid-surface forms curve C_α which represents the meridian. Curve C_α has, in point M, the centre of curvature O_α, where the distance between the centre of curvature O_α and point M is the principal curvature radius r_α.

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The second principal plane is perpendicular to the first principal plane, and the second principal curvature radius r_φ, which is determined by the distance between point M and the centre of curvature O_φ located on the axis of rotation lies here. The task is rotationally symmetric for all points lying on circle C_φ on the shell mid-surface. The distance of each point of the circle C_φ from the centre of curvature O_φ is r_φ. The principal curvature radius r_φ is a cone the base of which is the circle C_φ. The position of points on the circle C_φ is determined by the angle φ. For radius r of the circle C_φ , it holds that $r=r_\varphi{\cdot}\sin\,\alpha$.

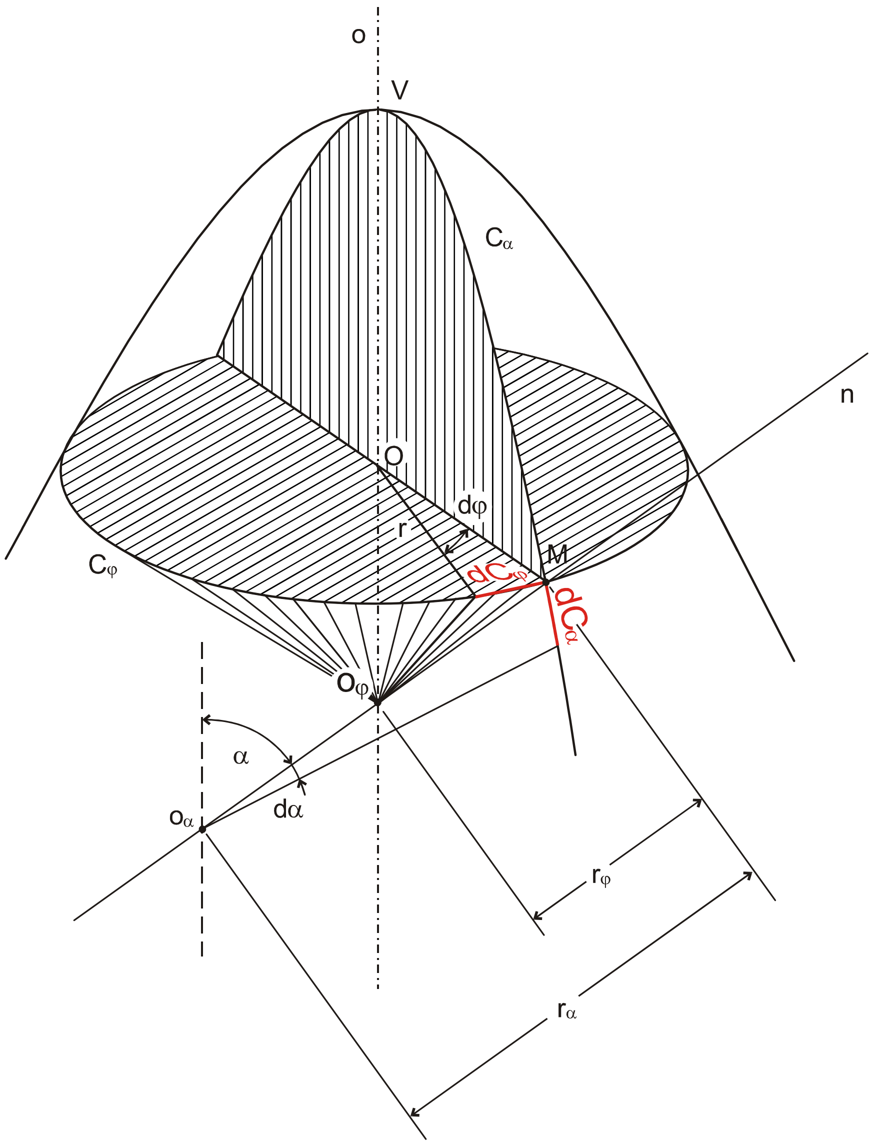

Fig. 1: Rotationally symmetric shell

The principal sections form a network of meridians and parallels on the mid-surface, the intersections of which are determined by angles α and φ. If we consider hydrostatic pressure as our only load, the internal forces may then be analysed using the equilibrium equations of the membrane state of an axially symmetric shell, see Fig. 2.

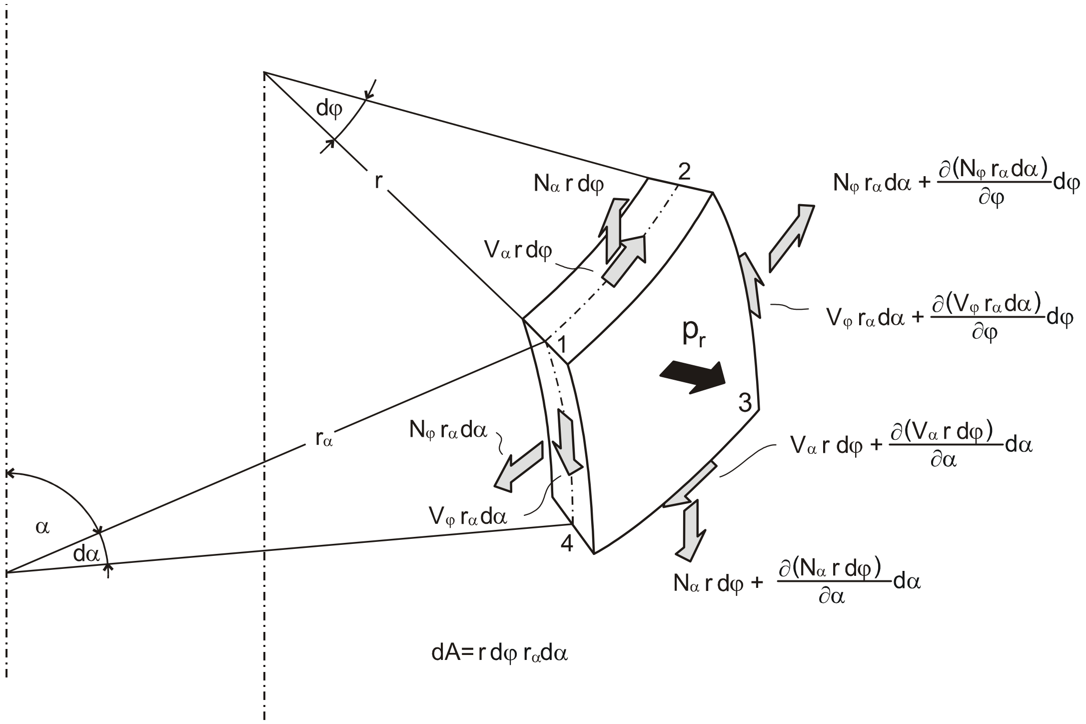

Fig. 2: Internal forces of the shell in membrane state

Hydrostatic pressure from the liquid is represented by the surface load p_r [Nm^{-2}]. Loading p_r evokes internal forces, which can be analysed using the conditions of equilibrium. The arrows in Fig. 2. represent internal forces N, V. By multiplying the membrane size (N_α, N_φ, V_α, V_φ) by the length of the segment on which it acts, we obtain the respective internal force. For example, the normal component N_α, acts on segment 12, and it corresponds to the force $N_\alpha \cdot r \cdot d\varphi$. Movement by $r_\alpha \cdot d\alpha$ in the direction of the meridian leads to change of N_α to the value $\frac{\partial N_\alpha}{\partial \alpha} d\alpha$, whereas by multiplying by the length of element 34, we obtain the relation for the normal force in the form

$$\left(N_\alpha + \frac{\partial N_\alpha}{\partial \alpha} d\alpha \right) \left(r \cdot d\varphi + \frac{\partial (r \cdot d\varphi)}{\partial \alpha} d\alpha \right). \tag{1}$$

Modifying (1) and neglecting small quantities of higher orders, we obtain

$$N_\alpha \cdot r \cdot d\varphi + \frac{\partial (N_\alpha \cdot r \cdot d\varphi)}{\partial \alpha} d\alpha . \tag{2}$$

From the moment equilibrium condition with regard to the centre of the element, and neglecting the members of the higher orders, we obtain

$$V_\alpha \cdot r \cdot d\varphi \cdot r_\alpha \cdot d\alpha - V_\varphi \cdot r_\alpha \cdot d\varphi \cdot r \cdot d\alpha = 0 , \tag{3}$$

from which it follows that $V_\alpha = V_\varphi = V$. Furthermore, let us express the force equilibrium condition in the direction of the normal. The resultant loading caused by the hydrostatic pressure of oil acting on the thin wall in the direction of its normal is

$$p_r \cdot dA = p_r \cdot r \cdot d\varphi \cdot r_\alpha \cdot d\alpha . \tag{4}$$

The resultant of membrane forces N_α in the direction of the normal upon neglecting the small quantities of higher orders is

$$N_{\alpha} \cdot r \cdot d\varphi \cdot d\alpha \, , \tag{5}$$

which is shown in Fig. 3, representing the vertical section passing through the axis of the shell.

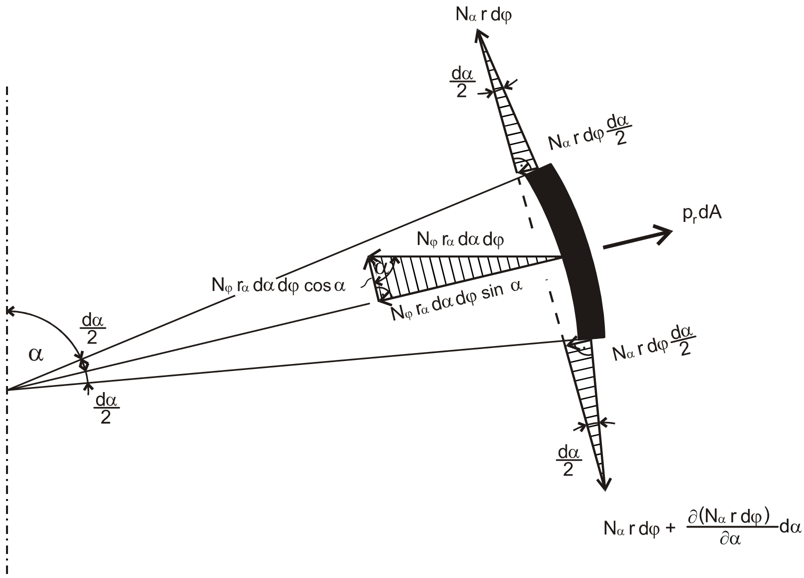

Fig. 3: Force equilibrium condition in the direction of the normal in the vertical section

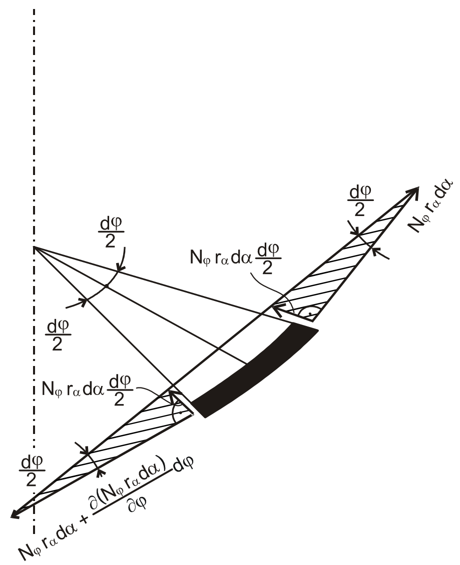

Fig. 4: Force equilibrium condition in the direction of the normal in the horizontal section

The projection of membrane forces N_φ in the direction of the normal is displayed using sections perpendicular to the axis of rotation, see Fig. 4. By adding the two normal forces $N_\varphi \cdot r_\alpha \cdot d\alpha \cdot \sin(d\varphi/2)$ and substituting $\sin(d\varphi/2) \approx d\varphi/2$, we can express the force in the horizontal plane as $N_\varphi \cdot r_\alpha \cdot d\alpha \cdot d\varphi$, see Fig. 4. and Fig. 3. Upon projecting this force in the direction of the normal, see Fig. 3, we obtain

$$N_{\varphi} \cdot r_{\alpha} \cdot d\alpha \cdot d\varphi \cdot \sin(\alpha) \, . \tag{6}$$

Adding (5), (6) and (4), we obtain the force equilibrium condition (7) in the direction of the normal.

$$N_\alpha \cdot r \cdot d\varphi \cdot d\alpha + N_\varphi \cdot r_\alpha \cdot d\alpha \cdot d\varphi \cdot \sin(\alpha) - p_r \cdot r \cdot d\varphi \cdot r_\alpha \cdot d\alpha = 0 . \qquad (7)$$

Substituting $r=r_\varphi \cdot \sin(\alpha)$ into equation (7), we obtain (8).

$$N_\alpha \cdot r_\varphi + N_\varphi \cdot r_\alpha = p_r \cdot r_\varphi \cdot r_\alpha . \qquad (8)$$

Modifying equation (8), we obtain the final form (9) of the equation.

$$\frac{N_\alpha}{r_\alpha} + \frac{N_\varphi}{r_\varphi} = p_r . \qquad (9)$$

3 THE RELIABILITY CONDITION OF THIN-WALLED CIRCULAR SHELLS

A typical example of vertical cylindrical steel storage shells are tanks for the storage of crude oil, which are made of thin plates. The dominant load case of the cylindrical tank is the load on the inner surface of the shell due to hydrostatic pressure of oil which loads the shell constantly in parallel circles. For thin-walled circular cylindrical tanks, it holds that α=90°, $r=r_\varphi \cdot \sin 90° = r_\varphi$, $r_\alpha=\infty$, $\varphi \in \langle 0; 180° \rangle$, $0<r_\varphi=r<\infty$, see Fig. 5.

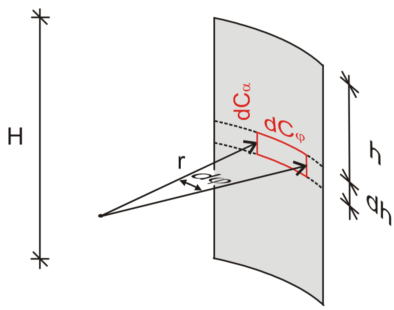

Fig. 5: Thin-walled cylindrical shell

Substituting $r_\alpha=\infty$ and $r= r_\varphi$ into (9), we obtain the equation for a rotationally symmetric circular cylindrical tank as

$$N_\varphi = p_r \cdot r , \qquad (10)$$

N_φ is the internal force evoked by the hydrostatic pressure caused by the oil which acts on dC_α, the other internal forces N_α, V_α, V_φ are equal to zero; it results from the equilibrium conditions. Surface loading p_r [Nm^{-2}] at the depth h is due to hydrostatic pressure of oil and is of magnitude:

$$p_r = \rho \cdot g \cdot h , \qquad (11)$$

where ρ is the density of oil at temperature 20°, which can be considered as $880\, kgm^{-3}$ (globally listed values are in the interval 730 - 1000 kgm^{-3}), g is acceleration due to gravity, which, in our latitude (49°), is approximately g=9.81$m \cdot s^{-2}$.

Substituting (11) into (10), we obtain

$$N_\varphi = \rho \cdot g \cdot h \cdot r . \qquad (12)$$

The resultant force acting perpendicularly to the surface $t \cdot dC_\alpha$ (in the centre of segment 14 or 23) is

$$F = N_\varphi \cdot dC_\alpha . \qquad (13)$$

For reliable design and operation of the tank, the reserve resistance G must be greater than zero, i.e., the loading force F (action) is less than or equal to the plate resistance R:

$$G = R - F > 0\,, \tag{14}$$

where R is the resistance which can be calculated as

$$R = f_y \cdot t \cdot dC_\alpha\,, \tag{15}$$

where f_y is the yield strength. The condition of reliability (14) can then be written with regard to (13) and (15) as

$$G = f_y \cdot t - \rho \cdot g \cdot h \cdot r \geq 0\,. \tag{16}$$

Equation (16) represents the equation for assessing the design reliability or service reliability of circular cylindrical tanks according to a number of standards, e.g., EEMUA 159 [5] or API653 [6]. Reliability according to standards is provided mainly by low (safe) values of the permissible stress, however, a non-zero probability that failure will occur always exists, see, e.g., [1, 3].

4 RELIABILITY ANALYSIS

Variables in equation (16) are considered as random in probabilistic assessments of reliability. Failure occurs when inequality (16) is not fulfilled. Reliability of all courses can be evaluated using the reliability index β acc. to Cornell [7], which is defined as the reciprocal of the variation coefficient of the reserve reliability, and is determined assuming normality of distribution of G acc. to the relation:

$$\beta = \frac{\mu_G}{\sigma_G}\,, \tag{17}$$

where μ_G is the mean value and σ_G is the standard deviation of random variable G. Reliability index β is related to the failure probability P_f acc. to the relation $P_f = \Phi(-\beta)$, where Φ is the cumulative distribution function of the standardized Gaussian probability density function. Due to degradation phenomena, decrease in the thickness of the tank shell takes place during operation. This is frequently caused by corrosion. The effect of changes in plate thicknesses can be studied using (17) provided that random variables G, F, R have Gaussian probability density functions. Let us study the relationship between the nominal plate thickness t and value β (17). In (16) h = 22.3 - 0.3 = 22 m (depth of 0.3 m from the bottom). The distance 0.3 m from the bottom in the reliability condition (16) is taken into account because the calculation of internal forces (9) is inaccurate near the bottom or change in thicknesses in the connection of courses. The value of 0.3 m is listed in standards EEMUA 159 and API 653. Input random values f_y and t were considered with Gaussian probability density functions. Yield strength f_y was considered with mean value μ_{fy} = 393.5 MPa and standard deviation σ_{fy} = 25.4 MPa [8]. The thickness of the course t was introduced with mean value $\mu_t = t$, and standard deviation $\sigma_t = 0.04 \cdot t$ [8]. Random variables f_y and t are statistically independent.

The evaluation of β (17) was performed using 100 simulation runs of the Latin Hypercube Sampling method [7, 9]. As an illustrative example, let us consider the tank which is in operation near the village of Nelahozeves. The tank radius is 42.235 m, the fill height is 22.3 m, the tank height is H=24.0 m. The curves of β vs t for a selected set of values $\rho{\cdot}g{\cdot}h{\cdot}r$ are presented in Fig. 6. The solid line shows the curve for $\rho{\cdot}g{\cdot}h{\cdot}r$=9115 kNm^{-1}; corresponding to parameters ρ=1000 kgm^{-3}, g=9.81ms^{-2}, h=22 m, r=42.235 m. It is obvious that the reliability is sufficient for values corresponding to an actual tank of thickness t=39 mm. The values β for other intermediate factors ρ, g, h, r, t are interpolated from Fig. 6 analogously.

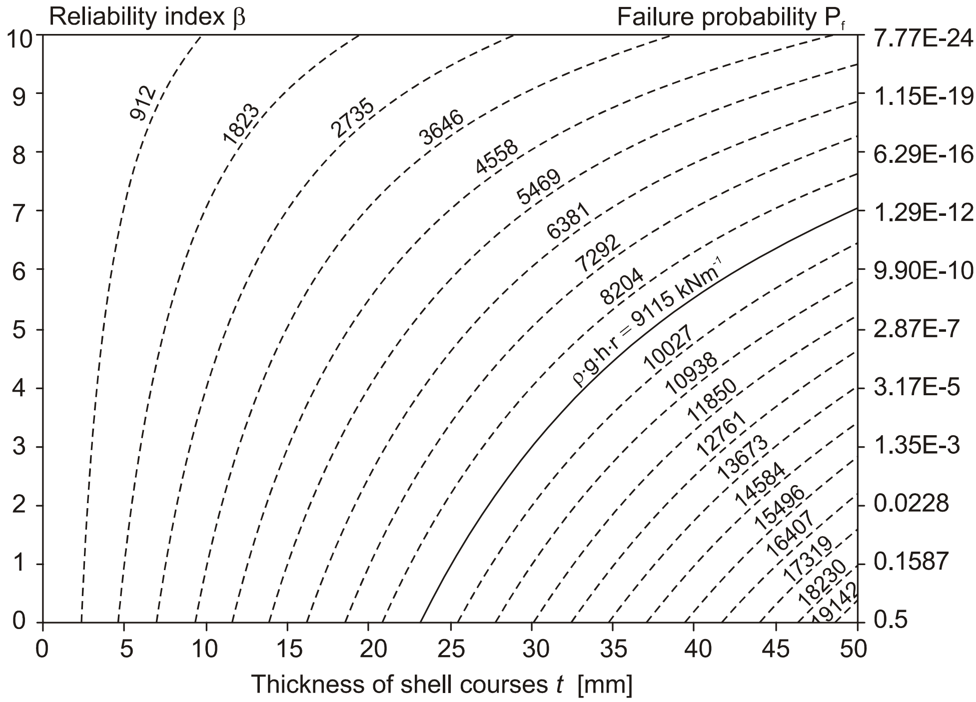

Fig. 6: Reliability index β vs thickness t

5 CONCLUSIONS

The methodology for the evaluation of reliability of thin-walled rotational symmetric cylindrical shells was demonstrated in the paper. The reliability was analysed using reliability index β. Fig. 6 shows how the value of β decreases with decreasing t. Decrease in thickness t is usually due to corrosion. For safe operation $\beta > \beta_t$, where the target value β_t is the normative value ensuring reliable operation of thin-walled structures. The generalization of the analysis of reliability evaluated acc. to $\rho{\cdot}g{\cdot}h{\cdot}r$ makes it possible to assess the reliability of technologically important facilities of power plants which are modelled as thin-walled rotationally symmetric cylindrical shells loaded along their perimeters by constant pressure perpendicular to the surface of the shell.

ACKNOWLEDGMENT

The article was elaborated within the framework of the project VG20132015109.

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